

1 DeMorgan's Laws

Use truth tables to show that $\neg(A \vee B) \equiv \neg A \wedge \neg B$ and $\neg(A \wedge B) \equiv \neg A \vee \neg B$. These two equivalences are known as DeMorgan's Laws.

2 Propositional Practice

Convert the following English sentences into propositional logic and the following propositions into English. State whether or not each statement is true with brief justification. Recall that \mathbb{N} refers to the set of natural numbers, \mathbb{Q} refers to the set of rational numbers, \mathbb{Z} refers to the set of integers, \mathbb{C} refers to the set of complex numbers, and $x \mid y$ denotes that x divides y .

- (a) There is a real number which is not rational.
- (b) All integers are natural numbers or are negative, but not both.
- (c) If a natural number is divisible by 6, it is divisible by 2 or it is divisible by 3.
- (d) $(\forall x \in \mathbb{R}) (x \in \mathbb{C})$
- (e) $(\forall x \in \mathbb{Z}) (((2 \mid x) \vee (3 \mid x)) \implies (6 \mid x))$
- (f) $(\forall x \in \mathbb{N}) ((x > 7) \implies ((\exists a, b \in \mathbb{N}) (a + b = x)))$

3 Preserving Set Operations

For a function f , define the image of a set X to be the set $f(X) = \{y \mid y = f(x) \text{ for some } x \in X\}$. Define the inverse image or preimage of a set Y to be the set $f^{-1}(Y) = \{x \mid f(x) \in Y\}$. Prove the following statements, in which A and B are sets.

Note: You will complete an extended version of this problem in HW0, in which you'll demonstrate that inverse images preserve set operations, but images typically do not.

Hint: For sets X and Y , $X = Y$ if and only if $X \subseteq Y$ and $Y \subseteq X$. To prove that $X \subseteq Y$, it is sufficient to show that $(\forall x) ((x \in X) \implies (x \in Y))$.

(a) $f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$.

(b) $f(A \cup B) = f(A) \cup f(B)$.