

1 Contraposition

Prove the statement "if $a + b < c + d$, then $a < c$ or $b < d$ ".

2 Numbers of Friends

Prove that if there are $n \geq 2$ people at a party, then at least 2 of them have the same number of friends at the party. Assume that friendships are always reciprocated: that is, if Alice is friends with Bob, then Bob is also friends with Alice.

(Hint: The Pigeonhole Principle states that if n items are placed in m containers, where $n > m$, at least one container must contain more than one item. You may use this without proof.)

3 Prime Form

Prove that every prime number $m > 3$ is either of the form $6k + 1$ or $6k - 1$ for some integer k .