

1. [True or False] Mark each of the following "True" if it is a valid logical equivalence, or "False" otherwise.

(a) $P \implies Q \equiv P \vee \neg Q$

(b) $P \implies Q \equiv (\neg P \implies \neg Q)$

(c) $P \implies Q \equiv (Q \wedge P) \vee \neg P$

2. [True or False] Let $P(x)$ and $Q(x)$ be a propositions about an integer x , and suppose you want to prove the theorem $\forall x, (P(x) \implies Q(x))$. Mark each of the following proof strategies "True" if it would be a valid way to proceed with such a proof, or "False" otherwise.

(a) Find an x such that $Q(x)$ is true or $P(x)$ is false.

(b) Show that, for every x , if $Q(x)$ is false then $P(x)$ is false.

(c) Assume that there exists an x such that $P(x)$ is false and $Q(x)$ is false and derive a contradiction.

(d) Assume that there exists an x such that $P(x)$ is true and $Q(x)$ is false and derive a contradiction.

3. [Proof] Suppose you have a rectangular array of pebbles, where each pebble is either red or blue. Suppose that for every way of choosing one pebble from each column, there exists a red pebble among the chosen ones. Prove that there must exist an all-red column.