## 1 Rolling Dice

(a) Suppose we are rolling a fair 6-sided die. What is the expected number of times we have to roll before we roll a 6? What is the variance?

(b) Suppose we have two independent, fair $n$-sided dice labeled Die 1 and Die 2. If we roll the two dice until the value on Die 1 is smaller than the value on Die 2, what is the expected number of times that we roll? What is the variance of the number of times that we rol?

## 2 Alternating Technicians

A faulty machine is repeatedly run and on each run, the machine fails with probability $p$ independent of the number of runs. Let the random variable $X$ denote the number of runs until the first failure. Now, two technicians are hired to check on the machine every run. They decide to take turns checking on the machine every run. What is the probability that the first technician is the first one to find the machine broken? (Your answer should a be closed-form expression.)

## 3 Student Request Collector

After a long night of debugging, Alvin has just perfected the new homework party/office hour queue system. CS 70 students sign themselves up for the queue, and TAs go through the queue, resolving requests one by one. Unfortunately, our newest TA (let's call him TA Bob) does not understand how to use the new queue: instead of resolving the requests in order, he always uses the Random Student button, which (as the name suggests) chooses a random student in the queue for him. To make matters worse, after helping the student, Bob forgets to click the Resolve button, so the student still remains in the queue! For this problem, assume that there are $n$ total students in the queue.

(a) Suppose that Bob has already helped $k$ students. What is the probability that the Random Student button will take him to a student who has not already been helped?

(b) Let $X_i^r$ be the event that TA Bob has not helped student $i$ after pressing the Random Student button a total of $r$ times. What is $\mathbb{P}[X_i^r]$? Assume that the results of the Random Student button are independent of each other. Now use the inequality $1 - x \leq e^{-x}$ to upper bound this probability.

(c) Let $T_r$ represent the event that TA Bob presses the Random Student button $r$ times, but still has not been able to help all $n$ students. (In other words, it takes TA Bob longer than $r$ Random Student button presses before he manages to help every student). What is $T_r$ in terms of the events $X_i^r$?

*Hint*: Events are subsets of the probability space $\Omega$, so you should be thinking of set operations.

(d) Using your answer for the previous part, what is an upper bound for $\mathbb{P}[T_r]$?

(e) Now let $r = \alpha n \ln n$. What is an upper bound for $\mathbb{P}[X_i^r]$?

(f) Calculate an upper bound for $\mathbb{P}[T_r]$ using the same value of $r$ as before. (This is more formally known as a bound on the tail probability of the distribution of button presses required to help every student.)

(g) What value of $r$ do you need to bound the tail probability by $1/n^2$? In other words, how many button presses are needed so that the probability that TA Bob has not helped every student is at most $1/n^2$?