1 Working with the Law of Large Numbers

(a) A fair coin is tossed multiple times and you win a prize if there are more than 60% heads. Which number of tosses would you prefer: 10 tosses or 100 tosses? Explain.

(b) A fair coin is tossed multiple times and you win a prize if there are more than 40% heads. Which number of tosses would you prefer: 10 tosses or 100 tosses? Explain.

(c) A fair coin is tossed multiple times and you win a prize if there are between 40% and 60% heads. Which number of tosses would you prefer: 10 tosses or 100 tosses? Explain.

(d) A fair coin is tossed multiple times and you win a prize if there are exactly 50% heads. Which number of tosses would you prefer: 10 tosses or 100 tosses? Explain.

2 Vegas

On the planet Vegas, everyone carries a coin. Many people are honest and carry a fair coin (heads on one side and tails on the other), but a fraction p of them cheat and carry a trick coin with heads on both sides. You want to estimate p with the following experiment: you pick a random sample of n people and ask each one to flip his or her coin. Assume that each person is independently likely to carry a fair or a trick coin.

1. Given the results of your experiment, how should you estimate p?
   (*Hint:* Construct an (unbiased) estimator for p such that $E[\hat{p}] = p$.)

2. How many people do you need to ask to be 95% sure that your answer is off by at most 0.05?

3 Dice

In this problem, let $X_1, X_2, \ldots X_n$ each denote the outcomes of standard six-sided dice rolls. Let A denote the average of the outcomes $(\sum_{i=1}^{n} X_i)/n$.

(a) For $n = 100$, find some a and b such that A is in the interval $[a, b]$ with probability at least 90% (Don't use trivial intervals like [1, 6]).

(b) For $n = 30$, find a lower bound on $\Pr[3 \le A \le 4]$.

(c) Find the minimum n for which you can guarantee that A is within the range [3, 4] with at least 99% probability.