1 Stable Matching

Consider the set of jobs $J = \{1, 2, 3\}$ and the set of candidates $C = \{A, B, C\}$ with the following preferences.

Jobs	Candidates
1	A > B > C
2	B > A > C
3	A > B > C

Candidates	Jobs
A	2 > 1 > 3
B	1 > 3 > 2
C	1 > 2 > 3

Run the traditional propose-and-reject algorithm on this example. How many days does it take and what is the resulting pairing? (Show your work.)

2 Propose-and-Reject Proofs

Prove the following statements about the traditional propose-and-reject algorithm.

(a) In any execution of the algorithm, if a candidate receives a proposal on day i, then she receives some proposal on every day thereafter until termination.

(b) In any execution of the algorithm, if a candidate receives no proposal on day i, then she receives no proposal on any previous day j, $1 \leq j < i$.

(c) In any execution of the algorithm, there is at least one candidate who only receives a single proposal. (Hint: use the parts above!)

3 Be a Judge

For each of the following statements about the traditional stable matching algorithm with jobs proposing, indicate whether the statement is True or False and justify your answer with a short 2-3 line explanation:

(a) There is a set of preferences for n jobs and n candidates for $n > 1$, such that in a stable matching algorithm execution every job ends up with its least preferred candidate.

(b) In a stable matching instance, if job J and candidate C each put each other at the top of their respective preference lists, then J must be paired with C in every stable pairing.

(c) In a stable matching instance with at least two jobs and two candidates, if job J and candidate C each put each other at the bottom of their respective preference lists, then J cannot be paired with C in any stable pairing.

(d) For every $n > 1$, there is a stable matching instance for n jobs and n candidates which has an unstable pairing in which every unmatched job-candidate pair is a rogue couple.

4 Universal Preference

Suppose that preferences in a stable matching instance are universal: all n jobs share the preferences $C_1 > C_2 > \cdots > C_n$ and all candidates share the preferences $J_1 > J_2 > \cdots > J_n$.

(a) What pairing do we get from running the algorithm with jobs proposing? Can you prove this happens for all n?

(b) What pairing do we get from running the algorithm with candidates proposing?

(c) What does this tell us about the number of stable pairings?