CS 70 Discrete Mathematics and Probability Theory
Spring 2020 DIS 2A

1 Stable Matching III

1. True or False?

   (a) If a candidate accidentally rejects a job she prefers on a given day, then the algorithm ends with a rogue couple.

   (b) The Propose-and-Reject Algorithm never produces a candidate-optimal matching.

   (c) If the same job is last on the preference list of every candidate, the job must end up with its least preferred candidate.

2. As you've seen from lecture, the jobs-proposing Propose-and-Reject Algorithm produces an employer-optimal stable matching. Let's see if the candidate have any way of improving their standing. Suppose exactly one of the candidates has the power to arbitrarily reject one proposal, regardless of which job she has on her string (if any). Construct an example that illustrates the following: for any $n \geq 2$, there exists a stable matching instance for which using this power helps **every** candidate, i.e. every candidate gets a better job than she would have gotten under the jobs-proposing Propose-and-Reject Algorithm.

2 Examples or It's Impossible

Determine if each of the situations below is possible with the traditional propose-and-reject algorithm. If so, give an example with at least 3 jobs and 3 candidates. Otherwise, give a brief proof as to why it's impossible.

(a) Every job gets its first choice.

(b) Every candidate gets her first choice, even though her first choice does not prefer her the most.

(c) Every candidate gets her last choice.

(d) Two or more jobs can have the same optimal candidate.

(e) A job which is second on every candidate's list gets its last choice.