

1 Let's Talk Probability

- (a) When is $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$ true? What is the general expression for $\mathbb{P}(A \cup B)$ that is always true.
- (b) When is $\mathbb{P}(A \cap B) = \mathbb{P}(A) * \mathbb{P}(B)$ true? What is the general expression for $\mathbb{P}(A \cap B)$ that is always true.
- (c) If A and B are disjoint, does that imply they're independent?

2 Aces

Consider a standard 52-card deck of cards:

- (a) Find the probability of getting an ace or a red card, when drawing a single card.
- (b) Find the probability of getting an ace or a spade, but not both, when drawing a single card.
- (c) Find the probability of getting the ace of diamonds when drawing a 5 card hand.
- (d) Find the probability of getting exactly 2 aces when drawing a 5 card hand.
- (e) Find the probability of getting at least 1 ace when drawing a 5 card hand.
- (f) Find the probability of getting at least 1 ace or at least 1 heart when drawing a 5 card hand.

3 Balls and Bins

Throw n balls into n labeled bins one at a time.

- (a) What is the probability that the first bin is empty?

- (b) What is the probability that the first k bins are empty?

- (c) Let A be the event that at least k bins are empty. Notice that there are $m = \binom{n}{k}$ sets of k bins out of the total n bins. If we assume A_i is the event that the i^{th} set of k bins is empty. Then we can write A as the union of A_i 's.

$$A = \bigcup_{i=1}^m A_i.$$

Write the union bound for the probability A .

- (d) Use the union bound to give an upper bound on the probability A from part (c).

- (e) What is the probability that the second bin is empty given that the first one is empty?

- (f) Are the events that "the first bin is empty" and "the first two bins are empty" independent?

- (g) Are the events that "the first bin is empty" and "the second bin is empty" independent?