

1 Head Count

Consider a coin with $\mathbb{P}(\text{Heads}) = 2/5$. Suppose you flip the coin 20 times, and define X to be the number of heads.

- (a) Name the distribution of X and what its parameters are.
- (b) What is $\mathbb{P}(X = 7)$?
- (c) What is $\mathbb{P}(X \geq 1)$? Hint: You should be able to do this without a summation.
- (d) What is $\mathbb{P}(12 \leq X \leq 14)$?

2 Exploring the Geometric Distribution

Suppose $X \sim \text{Geometric}(p)$ and $Y \sim \text{Geometric}(q)$ are independent. Find the distribution of $\min\{X, Y\}$ and justify your answer.

3 The Memoryless Property

Let X be a discrete random variable which takes on values in \mathbb{Z}_+ . Suppose that for all $m, n \in \mathbb{N}$, we have $\mathbb{P}(X > m + n \mid X > n) = \mathbb{P}(X > m)$. Prove that X is a geometric distribution. Hint: In order to prove that X is geometric, it suffices to prove that there exists a $p \in [0, 1]$ such that $\mathbb{P}(X > i) = (1 - p)^i$ for all $i > 0$.

4 Cookie Jars

You have two jars of cookies, each of which starts with n cookies initially. Every day, when you come home, you pick one of the two jars randomly (each jar is chosen with probability $1/2$) and eat one cookie from that jar. One day, you come home and reach inside one of the jars of cookies, but you find that is empty! Let X be the random variable representing the number of remaining cookies in non-empty jar at that time. What is the distribution of X ?