

Due: Fri 1/24, 11:59 PM
Grace period until Sun 1/26 11:59PM

1 Homework Process and Study Group

Citing sources and collaborators are an important part of life, including being a student! We also want to understand what resources you find helpful and how much time homework is taking, so we can change things in the future if possible.

1. **What sources (if any) did you use as you worked through the homework?**
2. **If you worked with someone on this homework, who did you work with?** List names and student ID's. (In case of homework party, you can also just describe the group.)
3. **How did you work on this homework?** (For example, *I first worked by myself for 2 hours, but got stuck on problem 3, so I went to office hours. Then I went to homework party for a few hours, where I finished the homework.*)
4. **Roughly how many total hours did you work on this homework?**

2 Administrivia

- (a) Make sure you are on the course Piazza (for Q&A) and Gradescope (for submitting homeworks, including this one). Find and familiarize yourself with the course website. What is its homepage's URL?
- (b) Read the policies page on the course website. What is the percentage breakdown of how your grade is calculated?

3 Course Policies

Go to the course website and read the course policies carefully. Leave a followup in the Homework 0, Question 2 thread on Piazza if you have any questions. Are the following situations violations of course policy? Write "Yes" or "No", and a short explanation for each.

- (a) Alice and Bob work on a problem in a study group. They write up a solution together and submit it, noting on their submissions that they wrote up their homework answers together.

- (b) Carol goes to a homework party and listens to Dan describe his approach to a problem on the board, taking notes in the process. She writes up her homework submission from her notes, crediting Dan.
- (c) Erin finds a solution to a homework problem on a website. She reads it and then, after she has understood it, writes her own solution using the same approach. She submits the homework with a citation to the website.
- (d) Frank is having trouble with his homework and asks Grace for help. Grace lets Frank look at her written solution. Frank copies it onto his notebook and uses the copy to write and submit his homework, crediting Grace.
- (e) Heidi has completed her homework using \LaTeX . Her friend Irene has been working on a homework problem for hours, and asks Heidi for help. Heidi sends Irene her PDF solution, and Irene uses it to write her own solution with a citation to Heidi.

4 Use of Piazza

Piazza is incredibly useful for Q&A in such a large-scale class. We will use Piazza for all important announcements. You should check it frequently. We also highly encourage you to use Piazza to ask questions and answer questions from your fellow students.

- (a) Navigate to the "Index" Piazza post, where you can find links to most resources in the course. Write down the Piazza post number for the Note 0 Thread. (When you see $@x$ on Piazza, where x is a positive integer, then x is the post number of the linked post.)
- (b) Read the Piazza Etiquette section of the course policies and explain what is wrong with the following hypothetical student question: "Can someone explain the proof of Theorem XYZ to me?" (Assume Theorem XYZ is a complicated concept.)
- (c) When are the weekly posts released? Are they required reading?

5 \LaTeX

If you have ample time on your hands and would like to learn a new skill, you may be interested in learning \LaTeX ! \LaTeX is a document preparation system that puts mathematical formulae into nicely formatted documents. Using \LaTeX can help you organize your thought process and make lives easier for readers. We have provided some resources on the course website to help you get started with using \LaTeX . Feel free to ask questions on Piazza if you have any questions.

For this question, try to typeset the following formulas. This will give you some practice writing mathematical formulas properly. If you choose hand-write and scan your solutions, just write out the formulas by hand.

Hint: You may find the *amsmath* \LaTeX package helpful.

(a) $\forall x \exists y ((P(x) \wedge Q(x, y)) \implies x \leq \sqrt{y})$

$$(b) \sum_{i=0}^k i = \frac{k(k+1)}{2}$$

6 Set Operations

- \mathbb{R} , the set of real numbers
- \mathbb{Q} , the set of rational numbers: $\{a/b : a, b \in \mathbb{Z} \wedge b \neq 0\}$
- \mathbb{Z} , the set of integers: $\{\dots, -2, -1, 0, 1, 2, \dots\}$
- \mathbb{N} , the set of natural numbers: $\{0, 1, 2, 3, \dots\}$

- (a) Given a set $A = \{1, 2, 3, 4\}$, what is $\mathcal{P}(A)$ (Power Set)?
- (b) Given a generic set B , how do you describe $\mathcal{P}(B)$ using set comprehension notation? (Set Comprehension is $\{x \mid x \in A\}$.)
- (c) What is $\mathbb{R} \cap \mathcal{P}(A)$?
- (d) What is $\mathbb{R} \cap \mathbb{Z}$?
- (e) What is $\mathbb{N} \cup \mathbb{Q}$?
- (f) What kind of numbers are in $\mathbb{R} \setminus \mathbb{Q}$?
- (g) If $S \subseteq T$, what is $S \setminus T$?

7 Preserving Set Operations

For a function f , define the image of a set X to be the set $f(X) = \{y \mid y = f(x) \text{ for some } x \in X\}$. Define the inverse image or preimage of a set Y to be the set $f^{-1}(Y) = \{x \mid f(x) \in Y\}$. Prove the following statements, in which A and B are sets. By doing so, you will show that inverse images preserve set operations, but images typically do not.

Hint: For sets X and Y , $X = Y$ if and only if $X \subseteq Y$ and $Y \subseteq X$. To prove that $X \subseteq Y$, it is sufficient to show that $(\forall x) ((x \in X) \implies (x \in Y))$.

- (a) $f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$.
- (b) $f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$.
- (c) $f^{-1}(A \setminus B) = f^{-1}(A) \setminus f^{-1}(B)$.
- (d) $f(A \cup B) = f(A) \cup f(B)$.
- (e) $f(A \cap B) \subseteq f(A) \cap f(B)$, and give an example where equality does not hold.
- (f) $f(A \setminus B) \supseteq f(A) \setminus f(B)$, and give an example where equality does not hold.

8 Linear Algebra

$$\text{Let } A = \begin{bmatrix} 1 & -1 & 2 & 2 \\ -2 & 3 & -3 & -1 \\ 3 & -3 & 6 & 7 \end{bmatrix}, \text{ and } b = \begin{bmatrix} 1 \\ -3 \\ 3 \end{bmatrix}.$$

- (a) Do the columns of A span \mathbb{R}^3 ? Justify your answer.
- (b) Are the columns of A linearly independent? Justify your answer.
- (c) Describe all solutions to $Ax = b$ in parametric vector form. Do the same for $Ax = 0$.
- (d) Provide an example of each of the following:
 - (i) An invertible matrix
 - (ii) A matrix with linearly independent columns, but linearly dependent rows
 - (iii) A 2×2 matrix with two real eigenvalues that are different from each other
 - (iv) A 2×2 matrix with a single real eigenvalue and only a single eigenvector
 - (v) A system of 2 equations in 2 unknowns with no solutions
- (e) A new streaming service charges 5 dollars per month for students, and 10 dollars per month for everyone else. This month, the service had 55 users, and collected 425 dollars. Set up a system of linear equations, and find the number of students using the service this month.

9 Calculus

- (a) Compute a closed-form expression for the value of following summation:

$$\sum_{k=1}^{\infty} \frac{9}{2^k}$$

- (b) Use summation notation to write an expression equivalent to the following statement:

The sum of the first n consecutive odd integers, starting from 1

- (c) Compute the following integral:

$$\int_0^{\infty} \sin(t)e^{-t} dt$$

- (d) Find the maximum value of the following function and determine where it occurs:

$$f(x) = -x \cdot \ln x$$

10 Academic Integrity

Please write or type out the following pledge in print, and sign it.

I pledge to uphold the university's honor code: to act with honesty, integrity, and respect for others, including their work. By signing, I ensure that all written homework I submit will be in my own words, that I will acknowledge any collaboration or help received, and that I will neither give nor receive help on any examinations.